110. In acute-angled triangle ABC the following relationship holds

$$\left(\sum \frac{1}{\sqrt{s-a}}\right)^2 \leqslant (m_a + w_b + h_c) \cdot \frac{4R+r}{3sr^2}.$$

Solution. Let r_a, r_b, r_c and F denote the exradii and the area of the triangle respectively. We use the well-known inequality

$$h_a + h_b + h_c \geqslant 9r,$$

and the equality

$$r_a + r_b + r_c = 4R + r.$$

Hence by the Cauchy-Schwarz inequality

$$\begin{aligned}(m_a + w_b + h_c) \cdot \frac{4R+r}{3sr^2} &\geqslant (h_a + h_b + h_c) \cdot \frac{4R+r}{3sr^2} \\ &\geqslant 3 \cdot \frac{4R+r}{F} = 3 \cdot \frac{r_a + r_b + r_c}{F} \\ &= 3 \cdot \sum \frac{1}{s-a} \geqslant \left(\sum \frac{1}{\sqrt{s-a}}\right)^2,\end{aligned}$$

and we are done.

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